محاضرة (3)

2- Principle of Minimum Variance Unbiased Estimation

Among all estimators of θ that are unbiased, choose the one that has minimum variance. The resulting is called the minimum variance unbiased estimator (MVUE) of θ. I The most important result of this type for our purposes concerns estimating the mean μ of normal distribution.

<u>**Theorem**</u> : Let $X1, \ldots, Xn$ be a random sample from a normal distribution with parameters μ and $\sigma 2$. Then the estimator μˆ = X is the MVUE for ¯ μ. I In some situations, it is possible to obtain an estimator with small bias that would be preferred to the best unbiased estimator.

3- <u>EFFICIENCY</u>:

An estimator is said to be efficient if in the class of unbiased estimators it has minimum variance.

Example: Suppose we have some prior knowledge that the population from which we are about to sample is normal. The mean of this population is however unknown to us. Because it is normal we know that $\overline{X}$ and $\text{median}_{\text{sample}}$ are unbiased

$$E(\overline{X}) = \mu$$

$$E(md) = \mu$$

However, consider their variances

$$V(md) = \frac{\pi \sigma^2}{2n}$$

$$Var(\overline{X}) = \frac{\sigma^2}{n}$$

Clearly, $\overline{X}$ is the more efficient since it has the smaller variance.

4 – **SUFFICIENCY:**

We say that an estimator is sufficient if it uses all the sample information. The median, because it considers only rank, is not sufficient. The sample mean considers each member of the sample as well as its size, so is a sufficient statistic. Or, given the sample mean, the distribution of no other statistic can contribute more information about the population mean. We use the factorization theorem to prove sufficiency. If the likelihood function of a random variable can be factored into a part which has as its arguments only the statistic and the population parameter and a part which involves only the sample data, the statistic is sufficient.

5- **Consistency**

One desirable property of estimators is consistency. If we collect a large number of observations, we hope we have a lot of information about any unknown parameter θ, and thus we hope we can construct an estimator with a very small MSE. We call an estimator consistent if limn MSE(θ) = 0 which means that as the number of observations increase the MSE descends to 0. In our first example, we found if X1, . . . , Xn $\sim$ N(θ, 1), then the MSE of ¯x is 1/n. Since limn(1/n) = 0, ¯x is a consistent estimator of θ. Remark: To be specific we may call this “MSE−consistant”. There are other type of consistancy definitions that, say, look at the probability of the errors. They work better when the estimator do not have a variance. If X1, . . . , Xn $\sim$ Uni(0, θ), then δ(x) = ¯x is not a consistent estimator of θ. The MSE is (3n + 1)θ 2/(12n) and limn (3n + 1)θ 2 12n = θ 2 4 6= 0 so even if we had an extremely large number of observations, ¯x would probably not be close to θ. Our adjusted estimator δ(x) = 2¯x is consistent, however. We found the MSE to be θ 2/3n, which tends to 0 as n tends to infinity. This doesn’t necessarily mean it is the optimal estimator (in fact, there are other consistent estimators with MUCH smaller MSE), but at least with large samples it will get us close to θ.